

Goldstein 3.19 (a)

Finding the effective force is straight forward:

$$F'(r) = F(r) + \frac{l^2}{mr^3}$$
$$= -\frac{k}{r^2} \exp\left(-\frac{r}{a}\right) + \frac{l^2}{mr^3}$$

This gives the effective 1D eqm:

$$m\ddot{r} = -\frac{k}{r^2} \exp\left(-\frac{r}{a}\right) + \frac{l^2}{mr^3}$$

The effective potential of $F(r) = -\frac{k}{r^2} \exp\left(-\frac{r}{a}\right)$ requires an integral not so easy to evaluate, we make note that

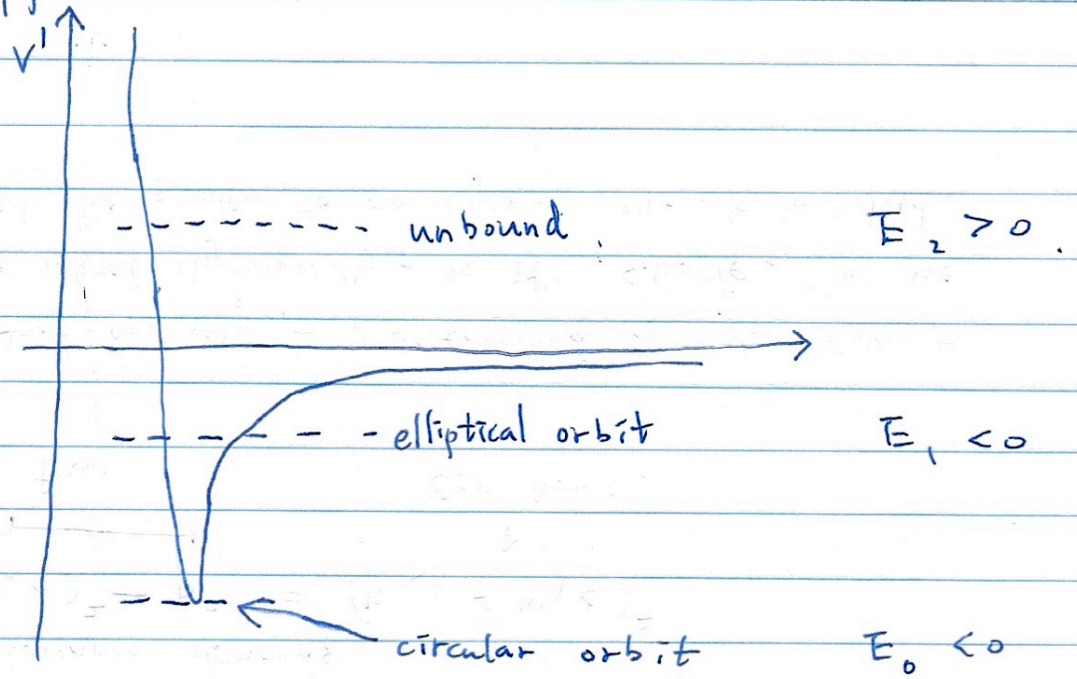
$$-\frac{d}{dr} \left[-\frac{k}{r} \exp\left(-\frac{r}{a}\right) \right]$$
$$= -\frac{k}{r^2} \exp\left(-\frac{r}{a}\right) + \frac{k}{ra} \exp\left(-\frac{r}{a}\right)$$

$$\approx -\frac{k}{r^2} \exp\left(-\frac{r}{a}\right) \quad \text{for } a \gg r.$$

We make this approximation, thus the effective potential is

$$V' = -\frac{k}{r} \exp\left(-\frac{r}{a}\right) + \frac{l^2}{2mr^2} \quad a \gg r.$$

For appropriate values of a, r, k, m, V' looks like



$$(2u - c) \cdot (u^2 + a^2) + (u^2 + c^2) \cdot (u^2 + a^2) = (u^2 + c^2) + (u^2 + a^2)$$

$$2u^2 + 2au - cu + 2a^2u - ca^2 + u^4 + cu^2 + a^2u^2 + c^2u^2 + a^2c^2 = u^4 + cu^2 + a^2u^2 + c^2u^2 + a^2c^2$$

$$2u^2 + 2au - cu + 2a^2u - ca^2 = 0$$

$$(2u^2 + 2au - cu + 2a^2u - ca^2) = (u^2 + cu^2 + a^2u^2 + c^2u^2 + a^2c^2) - (u^2 + cu^2 + a^2u^2 + c^2u^2 + a^2c^2)$$

$$2u^2 + 2au - cu + 2a^2u - ca^2 = 0$$

approx 12