Goldstein 3.19 (a)

Finding the effective force is straightforward:

$$F'(r) = F(r) + \frac{l^2}{mr^3}$$

$$= -\frac{k}{r^2}\exp\left(-\frac{r}{a}\right) + \frac{l^2}{mr^3}$$

This gives the effective 1D eqm:

$$\boxed{m\ddot{r} = -\frac{k}{r^2}\exp\left(-\frac{r}{a}\right) + \frac{l^2}{mr^3}}$$

The effective potential of $F(r) = -\frac{k}{r^2}\exp(-\frac{r}{a})$ requires an integral not so easy to evaluate, we make note that

$$-\frac{\partial}{\partial r}\left[-\frac{k}{r}\exp\left(-\frac{r}{a}\right)\right]$$

$$= -\frac{k}{r^2}\exp\left(-\frac{r}{a}\right) + \frac{k}{ra}\exp\left(-\frac{r}{a}\right)$$

$$\simeq -\frac{k}{r^2}\exp\left(-\frac{r}{a}\right) \quad \text{for } a \gg r.$$

We make this approximation, thus the effective potential is

$$V' = -\frac{k}{r}\exp\left(-\frac{r}{a}\right) + \frac{l^2}{2mr^2} \qquad a \gg r.$$

For approprіate values of a, r, k, m, V' looks like